CHAPTER 2
MATHEMATICAL INDUCTION AND THE BINOMIAL THEOREM

In this chapter we will investigate a method of proof called mathematical induction and then use mathematical induction to verify the binomial theorem for all positive integral values of n. We will also consider this theorem for fractional and negative values of n.

MATHEMATICAL INDUCTION

Mathematical induction is a proof that follows the idea that if we have a stairway of infinite steps and if we know we can take the first step and also that we can take a single step from any other step, we can, by taking steps one at a time, climb the stairway.

This proof is separated into two parts. First, we prove we can take the first step. Second, we assume we can reach a particular step, then we prove we can take one step from that particular step; therefore, we can climb the stairway.

To illustrate proof by mathematical induction we will prove that the sum of consecutive even integers in a series is equal to $n(n+1)$, where n represents the number of terms. The series is

$$2 + 4 + 6 + \cdots$$

We want to prove that

$$2 + 4 + 6 + \cdots + 2n = n(n + 1)$$

When n is 1 the formula yields

$$1(1 + 1)$$

$$= 2$$

This is true since the first term is shown to be 2. We check the formula when n is 2, although this is not necessary, and find that

$$2(2 + 1)$$

$$= 6$$

The sum of the first two terms is

$$2 + 4 = 6$$

which verifies the formula when n equals two.

We could verify the formula for as many values of n as we desire but this would not prove the formula for every value of n. We must now show that if the formula holds for the case where n equals K, then it holds for n equals $(K + 1)$. When n equals K in

$$2 + 4 + 6 + \cdots + 2n = n(n + 1)$$

we have

$$2 + 4 + 6 + \cdots + 2K = K(K + 1)$$

This we assume to be true.
Then, when n equals $(K + 1)$ we write

$$2 + 4 + 6 + \cdots + 2(K + 1) = (K + 1)(K + 1 + 1)$$

$$= (K + 1)(K + 2)$$

To show that this is true we add the $(K + 1)^{th}$ term to both sides of our assumed equality

$$2 + 4 + 6 + \cdots + 2K = K(K + 1)$$

which gives

$$2 + 4 + 6 + \cdots + 2K + 2(K + 1) = K(K + 1) + 2(K + 1)$$

In order to show that this is equal to

$$(K + 1)(K + 2)$$

we write

$$K(K + 1) + 2(K + 1) = (K + 1)(K + 2)$$

$$K^2 + K + 2K + 2 = (K + 1)(K + 2)$$

$$K^2 + 3K + 2 = (K + 1)(K + 2)$$

$$(K + 1)(K + 2) = (K + 1)(K + 2)$$

EXAMPLE: Use mathematical induction to prove that for all positive integral values,

$$1 + 3 + 5 + \cdots + (2n - 1) = n^2$$

SOLUTION: First step—Verify that this is true for $n = 1$. Substitute 1 for n and find that

$$n^2 = 1^2$$

$$= 1$$

which is the "sum" of the first term.

Second step—Assume the statement is true for $n = K$, then show it is true for $n = K + 1$, the next greater term than $n = K$.

When we assume the statement true for $n = K$, we may write

$$1 + 3 + 5 + \cdots + (2K - 1) = K^2$$

We now say that if $n = K + 1$ we may substitute $K + 1$ for n in the original statement and write

$$1 + 3 + 5 + \cdots + [2(K + 1) - 1] = (K + 1)^2$$

We must now verify that this is identical to adding the next term to both members of

$$1 + 3 + 5 + \cdots + (2K - 1) = K^2$$

We do this and find

$$1 + 3 + 5 + \cdots + (2K - 1) + (2K + 1) = K^2 + 2K + 1$$

We now verify that

$$K^2 + 2K + 1 = (K + 1)^2$$

Factoring the left member we find

$$K^2 + 2K + 1 = (K + 1)(K + 1)$$

$$= (K + 1)^2$$

which completes the verification.

We know that the original statement is true for $n = 1$. We proceed by letting $n = K = 1$. We find this is true. If we let $K = 2$, we find that

$$1 + 3 = 2^2$$

$$= 4$$

and when $K = 2$, we find that $K + 1 = 3$ then

$$1 + 3 + 5 = (K + 1)^2$$

$$= 3^2$$

$$= 9$$

Therefore, we may reason it is true for any positive integral value of n.

In general, to prove the validity of a given formula by the use of mathematical induction, we use two steps:

(1) Verify the given formula for $n = 1$.

(2) Assume the formula holds for $n = K$, then prove it is valid for $n = K + 1$ or the next larger value of n.

PROBLEMS: Prove by mathematical induction that the following series are valid for any positive integral value of n. Show steps (1) and (2).

1. $3 + 6 + 9 + \cdots + 3n = \dfrac{3n(n + 1)}{2}$

2. $1^2 + 2^2 + 3^2 + \cdots + n^2 = \dfrac{n(n + 1)(2n + 1)}{6}$

3. $1 + 4 + 7 + \cdots + (3n - 2) = \dfrac{n(3n - 1)}{2}$

ANSWERS:

1. (1) $n = 1$ then $\dfrac{3(1)(1 + 1)}{2} = 3$ (first term)

(2) $n = K$ then $\dfrac{3K(K + 1)}{2}$

$n = K + 1$ then $\dfrac{3(K + 1)(K + 1 + 1)}{2}$

$$= \frac{3(K + 1)(K + 2)}{2}$$

and

$$\frac{3K(K + 1)}{2} + 3(K + 1) = \frac{3K(K + 1) + 6(K + 1)}{2}$$

$$= \frac{3(K + 1)(K + 2)}{2}$$

2. (1) $n = 1$ then $\frac{1(1 + 1)(2 + 1)}{6} = 1$

(2) $n = K$ then $\frac{K(K + 1)(2K + 1)}{6}$

$n = K + 1$ then

$$\frac{(K + 1)(K + 1 + 1)(2K + 2 + 1)}{6}$$

$$= \frac{(K + 1)(K + 2)(2K + 3)}{6}$$

and

$$\frac{K(K + 1)(2K + 1)}{6} + (K + 1)^2$$

$$= \frac{K(K + 1)(2K + 1) + 6(K + 1)^2}{6}$$

$$= (K + 1)\left[\frac{2K^2 + 7K + 6}{6}\right]$$

$$= \frac{(K + 1)(K + 2)(2K + 3)}{6}$$

3. (1) $n = 1$ then $\frac{1(3 - 1)}{2} = 1$

(2) $n = K$ then $\frac{K(3K - 1)}{2}$

$n = K + 1$ then $\frac{(K + 1)(3K + 3 - 1)}{2}$

$$= \frac{(K + 1)(3K + 2)}{2}$$

and

$$\frac{K(3K - 1)}{2} + (3K + 1) = \frac{K(3K - 1) + 2(3K + 1)}{2}$$

$$= \frac{3K^2 - K + 6K + 2}{2}$$

$$= \frac{3K^2 + 5K + 2}{2}$$

$$= \frac{(K + 1)(3K + 2)}{2}$$

BINOMIAL THEOREM

The binomial theorem enables us to write any power of a binomial in the form of a sequence. This theorem is very useful in the study of probability and statistics. It is also useful in many other fields of mathematics.

EXPANSION

We use the binomial $(x + y)$ to indicate a general binomial, and to expand this binomial we raise it to increasing powers. That is, $(x + y)^n$ where n takes on the values 1, 2, 3, ... It is rather simple to raise the binomial to

$(x + y)^0 = 1,$ $\quad n = 0$

$(x + y)^1 = x + y,$ $\quad n = 1$

$(x + y)^2 = x^2 + 2xy + y^2,$ $\quad n = 2$

When we increase the value of n to 3, 4, 5, . . . we find it easier to use repeated multiplication. This results in

$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

and

$$(x + y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$$

If we consider the expansion $(x + y)^5$, we may determine the following:

(1) There are $n + 1$ terms in the expansion.

(2) x is the only variable in the first term and y is the only variable in the last term.

(3) In both the first and last term the exponent is n.

(4) As we move from left to right the exponent of x decreases by one and the sum of the exponents of each term is equal to n.

(5) The numerical coefficient of each term is determined from the term which precedes it by using the rule: The product of the exponent of x and the numerical coefficient, divided by the number which designates the position of the term, gives the value of the coefficient.

(6) There is symmetry about the middle term or terms of the numerical coefficients.

EXAMPLE: Write the expansion of $(x + y)^6$.

SOLUTION: (1) We know there are $n + 1$ or 7 terms.

(2) and (3) The first term is x^6 and the last term is y^6.

(4) The terms with their exponents but without their coefficients are

$$x^6 + x^5y + x^4y^2 + x^3y^3 + x^2y^4 + xy^5 + y^6$$

The numerical coefficient of the second term (determined by the first term) is

$$\frac{6 \cdot 1}{1} = 6$$

Then, we have $x^6 + \underline{6x^5y}$

The numerical coefficient of the third term (determined by the second term, $6x^5y$) is

$$\frac{5 \cdot 6}{2} = 15$$

Then, we have

$$x^6 + 6x^5y + \underline{15x^4y^2}$$

The numerical coefficient of the fourth term is

$$\frac{15 \cdot 4}{3} = 20$$

Then, we have $x^6 + 6x^5y + 15x^4y^2 + \underline{20x^3y^3}$

This process is continued to find

$$x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$$

EXAMPLE: Write the expansion of $(2x - 3y)^4$

SOLUTION: In this case we consider $(2x - 3y)^4$ to be

$$[(2x) + (-3y)]^4$$

We write the terms without coefficients as

$$(2x)^4 + (2x)^3(-3y) + (2x)^2(-3y)^2$$
$$+ (2x)(-3y)^3 + (-3y)^4$$

then determine the numerical coefficients as

$$(2x)^4 + 4(2x)^3(-3y) + 6(2x)^2(-3y)^2$$
$$+ 4(2x)(-3y)^3 + (-3y)^4$$

and carry out the multiplication indicated to find

$$16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4$$

EXAMPLE: Evaluate $(1 + 0.05)^5$ to the nearest hundredth.

SOLUTION: Write

$$(1 + 0.05)^5 = 1^5 + 5(1)^4(0.05)$$
$$+ 10(1)^3(0.05)^2 + 10(1)^2(0.05)^3$$
$$+ 5(1)^1(0.05)^4 + (0.05)^5$$

Term by term the values are

$$1^5 = 1$$

$$5(1)^4(0.05) = 0.25$$

$$10(1)^3(0.05)^2 = 0.025$$

$$10(1)^2(0.05)^3 = 0.00125$$

$$5(0.05)^4 = 0.00003125$$

$$(0.05)^5 = 0.0000003125$$

We are concerned only with hundredths; therefore, we add only the first four terms and find the sum to be

$$1.27628$$

which rounds to

$$1.28$$

For all positive values of n, the expansion of a binomial $(x + y)^n$ may be accomplished by following the previous rules indicated; that is,

$$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)x^{n-2}y^2}{1 \cdot 2}$$
$$+ \frac{n(n-1)(n-2)x^{n-3}y^3}{1 \cdot 2 \cdot 3} + \cdots + y^n$$

PROBLEMS: Write the expansion of the following:

1. $(x + 3y)^3$
2. $(2x - y)^4$
3. $(n + 3)^5$

ANSWERS:

1. $x^3 + 9x^2y + 27xy^2 + 27y^3$

2. $16x^4 - 32x^3y + 24x^2y^2 - 8xy^3 + y^4$

3. $n^5 + 15n^4 + 90n^3 + 270n^2 + 405n + 243$

PROBLEMS: Evaluate the following to the nearest hundredth.

1. $(1 + 0.01)^6$

2. $(1.03)^5$ or $(1 + 0.03)^5$

ANSWERS:

1. 1.06

2. 1.16

GENERAL TERM OF $(x + y)^n$

We consider the general term of $(x + y)^n$ as the r^{th} term. When we expand $(x + y)^n$ we have

$$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)x^{n-2}y^2}{1 \cdot 2}$$

$$+ \frac{n(n-1)(n-2)x^{n-3}y^3}{1 \cdot 2 \cdot 3} + \cdots + y^n$$

Notice that if we consider the term

$$\frac{n(n-1)(n-2)(n-3)x^{n-4}y^4}{1 \cdot 2 \cdot 3 \cdot 4}$$

as the r^{th} term, it is really the fifth term. The coefficient is

$$\frac{n(n-1)(n-2)(n-3)}{1 \cdot 2 \cdot 3 \cdot 4}$$

which is really

$$\frac{n(n-1) \cdots (n-3)}{1 \cdot 2 \cdot 3 \cdot 4}$$

If this is the r^{th} term, then, in the numerator

$$n - 3 = n - (r - 2)$$

$$= n - r + 2$$

and in the denominator

$$4 = r - 1$$

therefore,

$$\frac{n(n-1)(n-2)(n-3)}{1 \cdot 2 \cdot 3 \cdot 4}$$

is equal to

$$\frac{n(n-1)(n-2) \cdots (n-r+2)}{1 \cdot 2 \cdot 3 \cdots (r-1)}$$

and the exponents are

$$x^{n-4}y^4$$

or

$$x^{n-(r-1)}y^{(r-1)} = x^{n-r+1}y^{r-1}$$

Therefore, the r^{th} term where $r = 1, 2, 3, \ldots$ is

$$\frac{n(n-1)(n-2)(n-3) \cdots (n-r+2)}{1 \cdot 2 \cdot 3 \cdot 4 \cdots (r-1)} x^{n-r+1}y^{r-1}$$

At this point, the binomial formula holds for all positive integral values of n. Later, we will prove this to be true.

EXAMPLE: Find the 4^{th} term in the expansion of $(x + y)^8$.

SOLUTION: Write

$$n = 8$$

$$r = 4$$

then,

$$\frac{n(n-1)(n-2)(n-3) \cdots (n-r+2)}{1 \cdot 2 \cdot 3 \cdot 4 \cdots (r-1)} x^{n-r+1}y^{r-1}$$

$$= \frac{8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3} x^{8-4+1}y^{4-1}$$

$$= \frac{8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3} x^5y^3$$

$$= 56x^5y^3$$

EXAMPLE: Find the 3^{rd} term in the expansion of $(a - 3b)^5$.

SOLUTION: Write

$$n = 5$$

$$r = 3$$

and let

$$x = a$$

and

$$y = -3b$$

Then, using the binomial formula, find that

$$\frac{n(n-1)(n-2)\cdots(n-r+2)}{1\cdot 2\cdot 3\cdots(r-1)}\, x^{n-r+1}y^{r-1}$$

$$= \frac{5\cdot 4}{1\cdot 2}(a)^3(-3b)^2$$

$$= 10a^3(-3b)^2$$

$$= 90a^3b^2$$

PROBLEMS: Find the indicated term of the following by using the binomial formula:

1. 4^{th} term of $(x + y)^9$
2. 9^{th} term of $(x + y)^{12}$
3. 3^{rd} term of $(a^2 + B^2)^6$
4. 5^{th} term of $(2x - 3y)^7$

ANSWERS:

1. $84x^6y^3$
2. $495x^4y^8$
3. $15(a^2)^4(B^2)^2$ or $15a^8B^4$
4. $35(2x)^3(-3y)^4$ or $22680x^3y^4$

EXPANSION OF $(x + y)^n$ WHEN n IS NEGATIVE OR FRACTIONAL

The expansion of $(x + y)^n$ when n is negative or fractional does not terminate and holds only if y is numerically less than x. This is known as the binomial series.

EXAMPLE: Expand $(x + y)^{-2}$ to four terms and simplify.

SOLUTION: Write

$$(x + y)^{-2} = x^{-2} - 2x^{-3}y + 3x^{-4}y^2 - 4x^{-5}y^3 + \cdots$$

$$= \frac{1}{x^2} - \frac{2y}{x^3} + \frac{3y^2}{x^4} - \frac{4y^3}{x^5} + \cdots$$

EXAMPLE: Expand $(x + y)^{\frac{1}{2}}$ to four terms and simplify.

SOLUTION: Write

$$(x + y)^{\frac{1}{2}} = x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}y$$

$$+ \left(-\frac{1}{8}\right)x^{-\frac{3}{2}}y^2 + \frac{1}{16}x^{-\frac{5}{2}}y^3 + \cdots$$

$$= x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}y - \frac{1}{8}x^{-\frac{3}{2}}y^2 \; .$$

$$+ \frac{1}{16}x^{-\frac{5}{2}}y^3 + \cdots$$

$$= x^{\frac{1}{2}} + \frac{y}{2x^{\frac{1}{2}}} - \frac{y^2}{8x^{\frac{3}{2}}} + \frac{y^3}{16x^{\frac{5}{2}}} + \cdots$$

EXAMPLE: Expand $(1 + y)^{-\frac{1}{2}}$ to four terms and simplify.

SOLUTION: Write

$$(1 + y)^{-\frac{1}{2}} = 1^{\frac{-1}{2}} + \left(-\frac{1}{2}\right)(1)^{\frac{-3}{2}}y + \frac{3}{8}(1)^{\frac{-5}{2}}y^2$$

$$+ \left(-\frac{5}{16}\right)(1)^{\frac{-7}{2}}y^3 + \cdots$$

$$= 1 - \frac{1}{2}y + \frac{3}{8}y^2 - \frac{5}{16}y^3 \cdots$$

The binomial expansion is a useful tool in determining a root of a number to a particular degree of accuracy.

EXAMPLE: Evaluate $\sqrt{23}$ to the nearest tenth.

SOLUTION: Write

$$\sqrt{23} = \sqrt{25 - 2}$$

$$= (25 - 2)^{\frac{1}{2}}$$

The choice of 25 and 2 is made because 25 is the nearest square to 23. Then,

$$(25 - 2)^{\frac{1}{2}} = (25)^{\frac{1}{2}} + \frac{1}{2}(25)^{-\frac{1}{2}}(-2)$$

$$+ \left(-\frac{1}{8}\right)(25)^{-\frac{3}{2}}(-2)^2 + \cdots$$

$$= 5 - \frac{1}{5} - \frac{1}{250} + \cdots$$

$$= 5 - 0.20 - 0.004$$

$$= 4.796$$

$$= 4.8$$

EXAMPLE: Evaluate $\sqrt[5]{35}$ to the nearest tenth.

SOLUTION:

$$\sqrt[5]{35} = \sqrt[5]{32 + 3}$$

$$= (32 + 3)^{\frac{1}{5}}$$

The choice of 32 and 3 is made because 32 is $(2)^5$ which is the nearest 5^{th} power to 35. Then,

$$(32 + 3)^{\frac{1}{5}} = (32)^{\frac{1}{5}} + \frac{1}{5}(32)^{-\frac{4}{5}}(3)$$

$$+ \left(-\frac{2}{25}\right)(32)^{-\frac{9}{5}}(3)^2 + \cdots$$

$$= 2 + \frac{3}{80} - \frac{9}{6400} + \cdots$$

$$= 2 + 0.037 - 0.001$$

$$= 2.036$$

$$= 2.04$$

$$\approx 2.0$$

This answer may be verified by raising 2.04 to the fifth power; that is,

$$(2.04)^5 = (2 + 0.04)^5$$

$$= 2^5 + 5(2)^4(0.04) + 10(2)^3(0.04)^2$$

$$+ 10(2)^2(0.04)^3 + \cdots$$

$$= 32 + 3.20 + 0.1280 + 0.00256 + \cdots$$

$$= 35.33$$

$$\approx 35$$

PROBLEMS: Evaluate the following to the nearest tenth.

1. $\sqrt{30}$
2. $\sqrt[4]{22}$

ANSWERS:

1. 5.5
2. 2.2

PROOF BY MATHEMATICAL INDUCTION

We have shown that the binomial theorem is indicated (for all positive integral values of n) by

$$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n - 1)x^{n-2}y^2}{1 \cdot 2}$$

$$+ \frac{n(n - 1)(n - 2)x^{n-3}y^3}{1 \cdot 2 \cdot 3} + \cdots$$

$$+ nxy^{n-1} + y^n$$

To prove this by mathematical induction we show the two steps of the previous proof. That is, when n equals 1 the formula yields $(x + y)$. This is obvious by inspection.

In step (2) we assume the formula is true for n equals K by writing

$$(x + y)^K = x^K + Kx^{K-1}y + \frac{K(K - 1)}{1 \cdot 2} x^{K-2}y^2$$

$$+ \frac{K(K - 1)(K - 2)x^{K-3}y^3}{1 \cdot 2 \cdot 3} + \cdots + Kxy^{K-1} + y^K \quad (1)$$

Then, when n = K + 1, we have

$$(x + y)^{K+1} = x^{K+1} + (K + 1)x^K y$$

$$+ \frac{K(K + 1)x^{K-1}y^2}{1 \cdot 2} + \frac{K(K + 1)(K - 1)x^{K-2}y^3}{1 \cdot 2 \cdot 3} + \cdots$$

$$+ (K + 1)xy^K + y^{K+1} \quad (2)$$

This is what we wish to verify. In equation (1), to obtain the (K + 1) term of $(x + y)^K$, we must multiply $(x+y)^K$ by (x + y) which gives $(x+y)^{K+1}$. We must also multiply the right side of this equation by (x + y) in order to maintain our equality. When we multiply the right side of equation (1) by (x + y), we have

$$(x + y)\left[x^K + Kx^{K-1}y + \frac{K(K - 1)}{1 \cdot 2} x^{K-2}y^2 + \frac{K(K - 1)(K - 2)}{1 \cdot 2 \cdot 3} x^{K-3}y^3 + \cdots + Kxy^{K-1} + y^K\right]$$

which gives

$$x\left[x^K + Kx^{K-1}y + \frac{K(K - 1)}{1 \cdot 2} x^{K-2}y^2 + \frac{K(K - 1)(K - 2)}{1 \cdot 2 \cdot 3} x^{K-3}y^3 + \cdots + Kxy^{K-1} + y^K\right]$$

$$+ y\left[x^K + Kx^{K-1}y + \frac{K(K - 1)}{1 \cdot 2} x^{K-2}y^2 + \frac{K(K - 1)(K - 2)}{1 \cdot 2 \cdot 3} x^{K-3}y^3 + \cdots + Kxy^{K-1} + y^K\right]$$

By carrying out the indicated multiplication and then combining terms we have

$$x^{K+1} + (K + 1)x^K y + \frac{K(K + 1)}{1 \cdot 2} x^{K-1}y^2$$

$$+ \frac{K(K+1)(K-1)}{1 \cdot 2 \cdot 3} x^{K-2}y^3 + \cdots + (K+1)xy^K + y^{K+1}$$

which is identical to equation (2) and the validity of the theorem is proved.

PASCAL'S TRIANGLE

When we expand $(x + y)^n$ for $n = 0, 1, 2, \ldots$ we find

$$(x + y)^0 = 1$$

$$(x + y)^1 = 1x + 1y$$

$$(x + y)^2 = 1x^2 + 2xy + 1y^2$$

$$(x + y)^3 = 1x^3 + 3x^2y + 3xy^2 + 1y^3$$

$$(x + y)^4 = 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$$

We could continue this indefinitely but for explanation purposes we will stop at the point where n = 4.

If we remove everything except the numerical coefficients of each term we have

n = 0,	1
n = 1,	1 1
n = 2,	1 2 1
n = 3,	1 3 3 1
n = 4,	1 4 6 4 1

whose border forms an isosceles triangle bounded by 1's on two sides. This triangle is named for Blaise Pascal who discovered it. Pascal's triangle gives the numerical coefficients of the expansion of a binomial.

Each row, after the first, is formed from the row above it and there are n + 1 terms in each row. A row is formed by writing 1 as the first term and then adding the two numbers above and nearest to the number desired; that is,

n = 0,	1
n = 1,	1 1
n = 2,	1 2 1
n = 3,	1 3 3 1
n = 4,	1 4 6 4 1
n = 5,	1 5 10 10 5 1

EXAMPLE: Find the coefficients of $(x + y)^n$ when n equals 7.

SOLUTION: Write

$n = 0,$								1								
$n = 1,$							1		1							
$n = 2,$						1		2		1						
$n = 3,$					1		3		3		1					
$n = 4,$				1		4		6		4		1				
$n = 5,$			1		5		10		10		5		1			
$n = 6,$		1		6		15		20		15		6		1		
$n = 7,$	1		7		21		35		35		21		7		1	

Notice there are $n + 1$ terms in each row.